Alternating Current (AC) Circuits

- We have been talking about DC circuits
  - Constant currents and voltages
  - Resistors
  - Linear equations
- Now we introduce AC circuits
  - Time-varying currents and voltages
  - Resistors, capacitors, inductors (coils)
  - Linear *differential* equations

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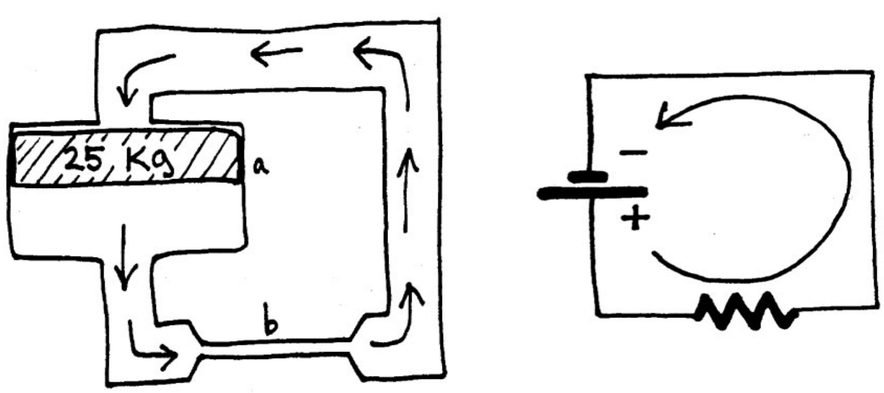
Recall water analogy for Ohm's law…

(a) Battery
(b) Resistor



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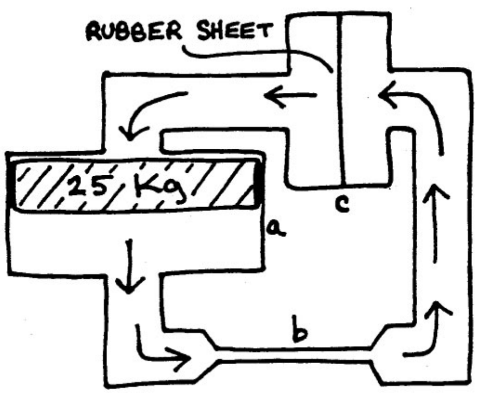
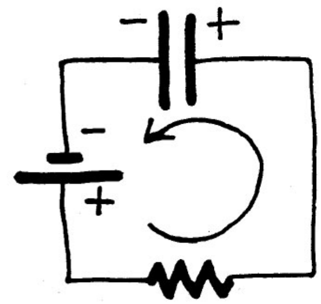
Now we add a steel tank with rubber sheet

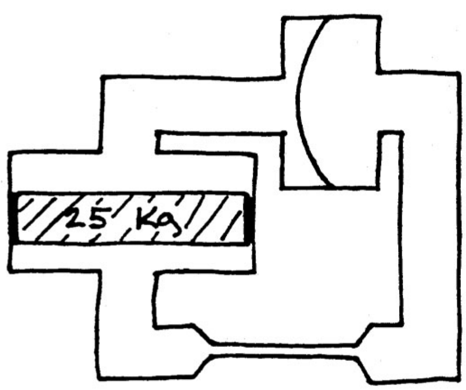
(a) Battery
(b) Resistor
(c) *Capacitor*

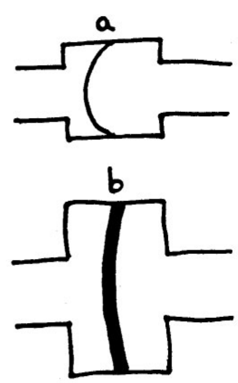




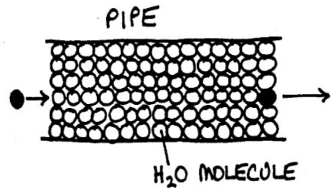
Water enters one side of the tank and leaves the other, distending but not crossing the sheet.

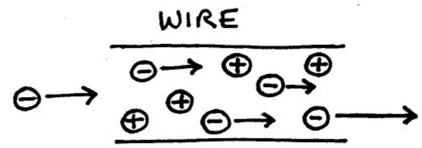
How to decrease *capacitance* of tank?
Make rubber sheet (a) smaller or (b) thicker, (or use stiffer rubber).



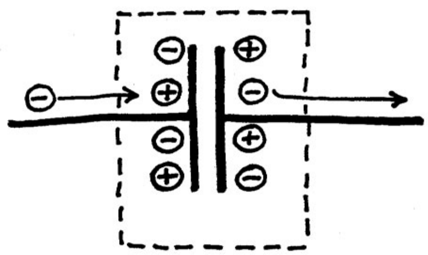


Charge, like water is practically incompressible,





but within a small volume (closely spaced plates) charge can enter one side and leave the other, without flowing across the space between.



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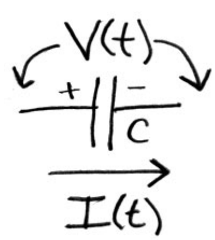
Basic Laws of Capacitance

- *Capacitance C* relates charge *Q* to voltage *V*

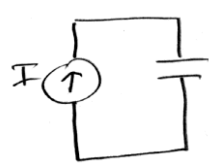
$$C = \frac{Q}{V}$$

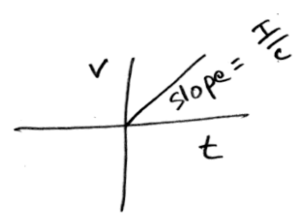


- Since $Q = \int I \, dt$,

$$V = \frac{1}{C}\int I \, dt$$

$$I = C\frac{dV}{dt}$$





- Inductance has units of *Farads*, F.

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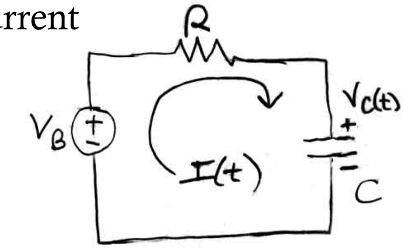
Charging a Capacitor with Battery V_B

- Voltage across resistor to find current

$$I(t) = \frac{V_B - V(t)}{R}$$



- Basic law of capacitor

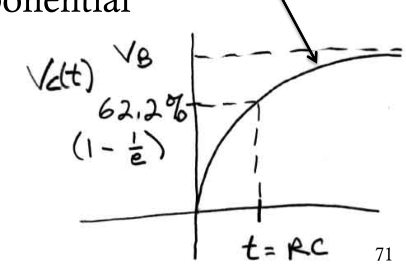
$$I(t) = C\frac{dV_C(t)}{dt}$$

- Differential Equation yields exponential

$$V_C(t) + RC\frac{dV_C(t)}{dt} = V_B$$

$$V_C(t) = V_B\left(1 - e^{-\frac{t}{RC}}\right)$$

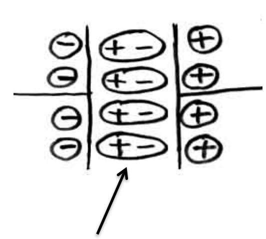
diminishing returns as cap becomes charged



What determines *capacitance* C?

- Area A of the plates
- Distance d between the plates
- *Permittivity* ε of the *dielectric* between plates.

$$C = \varepsilon\frac{A}{d}$$

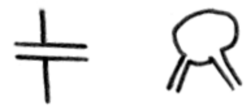


Alignment of dipoles within *dielectric* between plates increases capacitor's ability to store charge (capacitance).

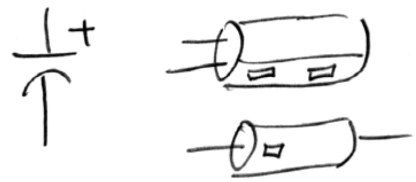
Permittivity of a vacuum $\varepsilon_0 \approx 8.8541 \times 10^{-12}\ \mathrm{F \cdot m^{-1}}$.

Types of Capacitors

- Disk (Ceramic) Capacitor
  - Non-polarized
  - Low leakage
  - High breakdown voltage
  - ~ 5pF – 0.1 μ F



- Electrolytic Capacitor
  - High leakage
  - Polarized
  - Low breakdown voltage
  - ~ 0.1 μ F – 10,000 μ F



- Supercapacitor (Electrochemical Double Layer)
  - New. Effective spacing between plates in nanometers.
  - Many Farads! May power cars someday.

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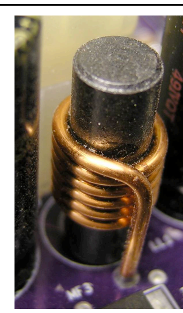
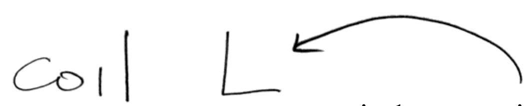
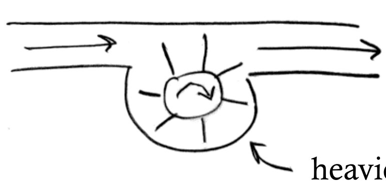
Inductor (coil)



- Water Analogy



inductance is like inertia/momentum of water in pipe with flywheel.



heavier flywheel (coil wrapped around iron core) adds to inertia/momentum.

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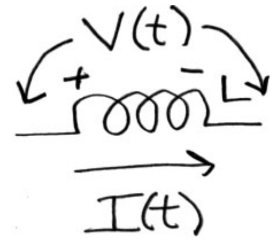
Basic Laws of Inductance

- *Inductance L* relates changes in the current to voltages induced by changes in the magnetic field produced by the current.

$$I = \frac{1}{L}\int V\,dt$$

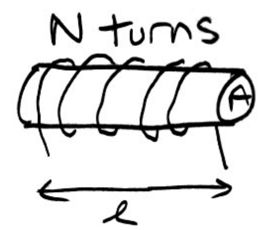
$$V = L\frac{dI}{dt}$$



- Inductance has units of *Henries,* H.

What determines *inductance L* ?

- Assume a solenoid (coil)
- Area A of the coil
- Number of turns N
- Length ℓ of the coil
- *Permeability* μ of the core



$$L = \mu\frac{N^2 A}{\ell}$$

Permeability of a vacuum $\mu_0 \approx 1.2566\times10^{-6}$ H·m^{-1}.

Energy Stored in Caps and Coils

- Capacitors store "potential" energy in electric field

$$E = \frac{1}{2} C V^2$$ independent of history

- Inductors store "kinetic" energy in magnetic field

$$E = \frac{1}{2} L\, I^2$$ independent of history

- Resistors don't store energy at all!

error in Scherz Eq. 2.42: missing "*dt*"

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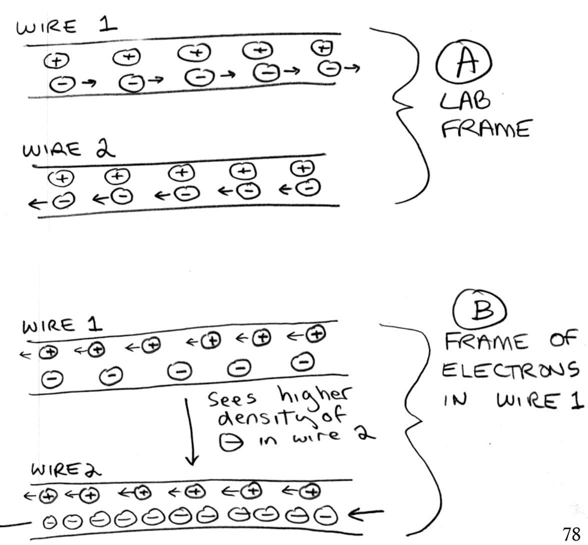
What is Magnetism?

- Lorenz Contraction $\ell = \ell_0 \sqrt{1 - v^2/c^2}$

Length ℓ of object observed in relative motion to the object is shorter than the object's length ℓ_0 in its own rest frame as velocity v approaches speed of light c.

Thus electrons in Wire 1 see Wire 2 as negatively charged and repel it: Magnetism!



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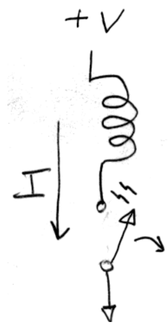
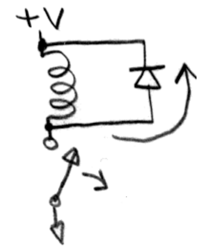
Generating Sparks

- What if you suddenly try to stop a current?



$$V = L\frac{dI}{dt}$$

goes to - ∞ when switch is opened.



use diode to shunt current, protect switch.

- Nothing changes instantly in Nature.
- Spark coil used in early radio (Titanic).
- Tesla patented the spark plug.

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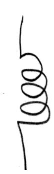
Symmetry of Electromagnetism

$$I = C\frac{dV}{dt} \qquad V = \frac{1}{C}\int I\,dt$$



$$V = L\frac{dI}{dt} \qquad I = \frac{1}{L}\int V\,dt$$

- Only difference is no magnetic monopole.

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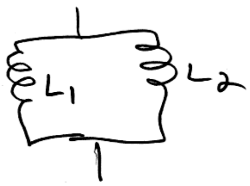
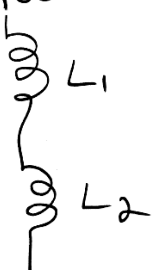
Inductance adds like Resistance

parallel



$$L_P = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$$

series



$$L_S = L_1 + L_2$$

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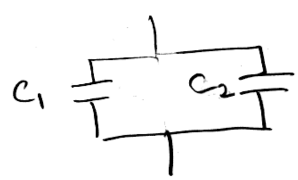
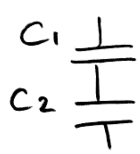
Capacitance adds like *Conductance*

parallel



$$C_P = C_1 + C_2$$

series



$$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

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