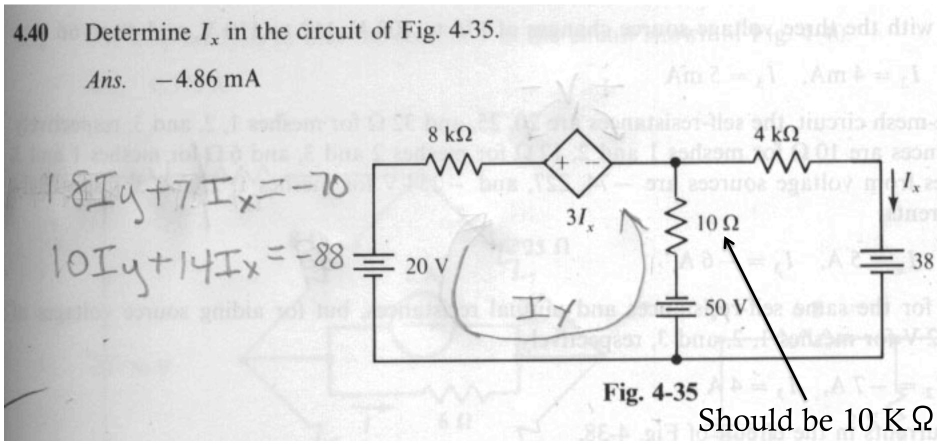
Bug in Homework

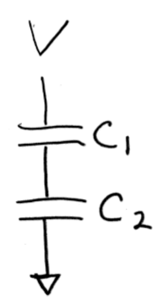


Distribution of charge and voltage on multiple capacitors

- To find the voltage on capacitors in series
  - Find total effective capacitance C_{Total}
  - Charge will be $Q_{\text{Total}} = C_{\text{Total}} V$
  - Same charge will be on all caps (Kirchoff's Current Law)
  - Voltage will be distributed inversely to capacitance

$$Q_{\text{Total}} = VC_{\text{Total}}$$
$$Q_{\text{Total}} = Q_1 = Q_2$$

$$V_1 = \frac{Q_1}{C_1} = \frac{Q_{\text{Total}}}{C_1} = \frac{C_{\text{Total}}}{C_1} V$$
$$V_2 = \frac{Q_2}{C_2} = \frac{Q_{\text{Total}}}{C_2} = \frac{C_{\text{Total}}}{C_2} V$$

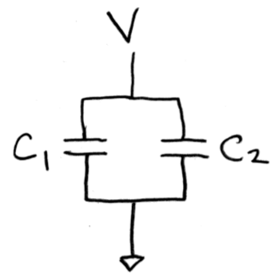


Distribution of charge and voltage on multiple capacitors

- To find the charge in capacitors in parallel
  - Find total effective capacitance C_{Total}
  - Charge will be $Q_{\text{Total}} = C_{\text{Total}} V$
  - Same voltage will be on all caps (Kirchoff's Voltage Law)
  - Q_{Total} will be divided proportionally to capacitance

$$Q_{\text{Total}} = VC_{\text{Total}} = Q_1 + Q_2$$
$$V = V_1 = V_2$$

$$Q_1 = VC_1$$
$$Q_2 = VC_2$$



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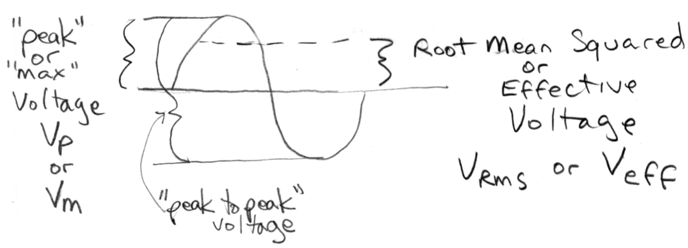
Definitions of Sinusoidal Amplitude



$$V_{Rms} = \frac{V_P}{\sqrt{2}}$$

- Average power = $V_{RMS} \times I_{RMS}$, when they are in-phase (resistor).
- Since $\sin^2 + \cos^2 = 1$, each has an average value of ½, and the V_{RMS} is V_P divided by $\sqrt{2}$.
- If V_{RMS} and I_{RMS} are 90° out-of-phase, the average power is zero, as they are in a coil or a capacitor, so the *LC* branch does not dissipate power and can "ring" forever.

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Let's prove those last statements

Just as we showed that when V_{RMS} and I_{RMS} are in-phase

$$\cos^2(\omega t) = \frac{1+\cos(\omega t)}{2}$$, whose average (DC) value of ½,

when V_{RMS} and I_{RMS} are 90° out-of-phase,

$$\cos(\omega t)\sin(\omega t) = \left(\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right)\left(\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right)$$

$$= \frac{e^{j2\omega t}-e^{-j2\omega t}+e^{0}-e^{0}}{4j}$$

$$= \frac{\sin(2\omega t)}{2}$$, whose average value is zero.

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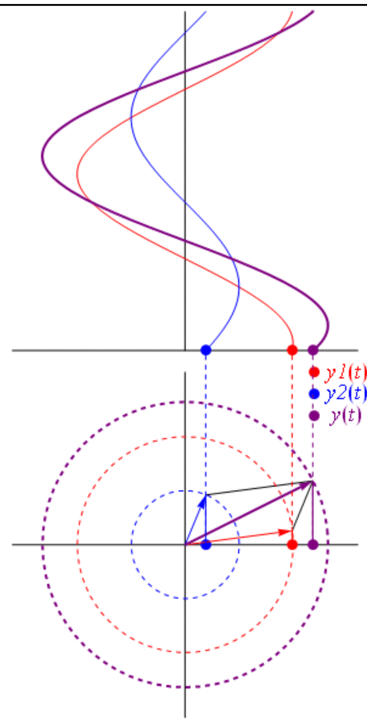

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Phasor Notation Ambiguity

- In electronics, complex exponentials are described unambiguously with shorthand notation

$$re^{j(\omega t+\theta)} \Rightarrow "r\angle\theta"$$

- Unfortunately when applied to real voltages and currents, the same notation is used variously to mean sine or cosine, peak or RMS. Thus, $A\angle\theta$ may mean

$$v(t)=\frac{A}{\sqrt{2}}\sin(\omega t+\theta)$$

or

$$v(t)=A\cos(\omega t+\theta)$$

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Phasor Notation Ambiguity (cont…)

- This ambiguity is allowed to continue because linear systems change only magnitude and phase.
- Thus a given network of coils, capacitors, and resistors will cause the same relative change in

$$v(t)=\frac{A}{\sqrt{2}}\sin(\omega t+\theta)$$

as it does in

$$v(t)=A\cos(\omega t+\theta)$$

so it doesn't matter which definition of $A\angle\theta$ you use for real signals, so long as you are consistent.

see my Phasor Notation Manifesto on website

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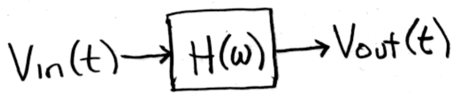
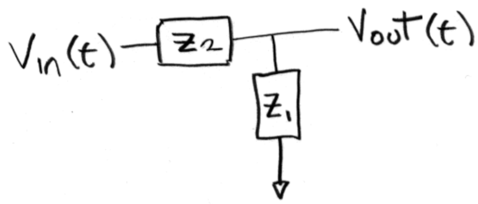
Systems modeled as Filters

- Has input and output signals (functions of time).
- Most electronic systems use voltages as inputs and outputs.
- Has a "transfer function" $H(\omega)$ in the frequency domain.
- $H(\omega)$ determines what happens to $e^{j\omega t}$, and thus any signal.



$$H(\omega)=\frac{V_{\text{out}}(t)}{V_{\text{in}}(t)}$$

- Consider system consisting of a voltage divider.
- Divider can now use complex impedances.



$$H(\omega)=\frac{Z_1}{Z_1+Z_2}$$

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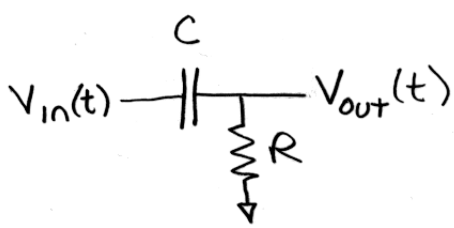
Example: RC High-Pass Filter



$$H(\omega)=\frac{V_{\text{out}}(t)}{V_{\text{in}}(t)}$$

$$H(\omega)=\frac{R}{R+\frac{1}{j\omega C}}=\frac{j\omega RC}{1+j\omega RC}$$

At high frequencies, acts like a piece of wire.

$$H(\omega)\cong 1,\quad \omega \gg \frac{1}{RC}$$

At low frequencies, attenuates and differentiates.

$$H(\omega)\cong j\omega RC,\quad \omega \ll \frac{1}{RC}$$

Key frequency is reciprocal of time constant *RC*.

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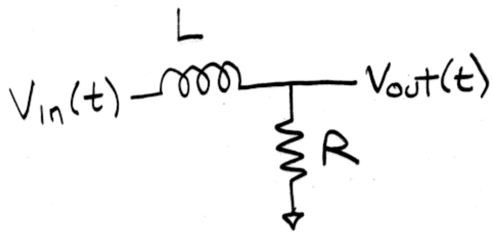
Example: LC Low-Pass Filter



$$H(\omega) = \frac{V_{\text{out}}(t)}{V_{\text{in}}(t)}$$

$$H(\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega \frac{L}{R}}$$

At low frequencies, acts like a piece of wire.

$$H(\omega) \cong 1, \quad \omega << \frac{R}{L}$$

At high frequencies, attenuates and integrates.

$$H(\omega) \cong \frac{R}{j\omega L}, \quad \omega >> \frac{R}{L}$$

Key frequency is reciprocal of time constant L/R.

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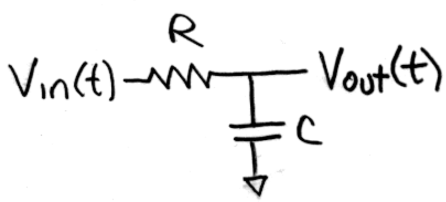
Example: RC Low-Pass Filter



$$H(\omega) = \frac{V_{\text{out}}(t)}{V_{\text{in}}(t)}$$

$$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

At low frequencies, acts like a piece of wire.
(*assuming no current at output*)

$$H(\omega) \cong 1, \quad \omega << \frac{1}{RC}$$

At high frequencies, attenuates and integrates.

$$H(\omega) \cong \frac{1}{j\omega RC}, \quad \omega >> \frac{1}{RC}$$

Key frequency is reciprocal of time constant RC.

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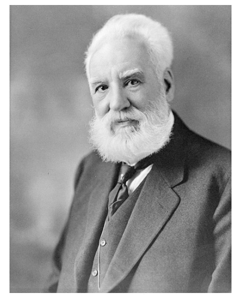
Decibels – a ratio

1 Bell = order of magnitude in power = 10dB

$dB = 10 \log\left(\frac{P_1}{P_2}\right)$

Alexander Graham Bell



e.g. if $P_1 = 100w$ and $P_2 = 1w$
we have a 20dB increase
from P_2 to P_1

$10 \log\left(\frac{100w}{1w}\right) = 20 dB$

↑ pure number

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Decibels are ratios of power, but we mainly use voltage

since power ∝ voltage²
10 × the voltage → 100 × power
or 20 dB

So a dynamic Range of 40 dB
↓

$\frac{V_{BIGGEST}}{V_{SMALLEST}} = 100 \quad \frac{P_{BIGGEST}}{P_{SMALLEST}} = 10^4$

(as in the mikes in our lab)

$$dB \equiv 10 \log\left(\frac{P_1}{P_2}\right) = 20 \log\left(\frac{V_1}{V_2}\right)$$

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Dynamic Range

- The useful range in a system, from smallest to largest signal, generally expressed in dB.
- *Analog Systems*: limited by the noise "floor" and by "clipping" at the maximum voltage.
- *Digital Systems*: limited by the number of bits representing the signal (as we'll see later).

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Absolute measures using decibels

dBm is power relative to 1 mW

so 1 mW = 0 dBm

1 W = 30 dBm

dBu is power into 600Ω

(professional low impedance audio)

0 dBu = 1 mW into 600Ω = .775 V

$\frac{V^2}{R} = 1\,mW \quad V = \sqrt{0.6} = .775\,V$

studios run at +4 dBu

$+4\,dBu = 10^{\frac{0.4}{2}} \times .775\,V = 1.23\,V$

divide by 2 because $V \propto \sqrt{P}$

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More absolute measures using decibels

dBV is voltage into high impedance (consumer audio)

$0\,\mathrm{dBV} = 1\,\mathrm{V}$

tends to run at $-10\,\mathrm{dBV}$

$-10\,\mathrm{dBV} = 10^{-\frac{1.0}{2}} \times 1\mathrm{V} = 0.316\,\mathrm{V}$

dBSPL (sound pressure level)

0dBSPL = threshold human hearing

our mike is 54 dB SPL

or $10^{5.4} \times$ threshold in power

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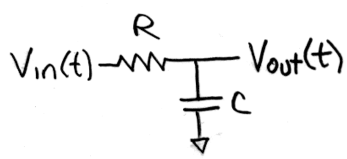
Magnitude and Phase of a Filter

Recall low-pass filter:



$$H(\omega) = \frac{V_{\text{out}}(t)}{V_{\text{in}}(t)} = \frac{1}{1 + j\omega RC}$$

$$V_{\text{out}}(t) = H(\omega)V_{\text{in}}(t)$$

stationary phasor changes magnitude and phase of signal

At *cut-off frequency*, $\omega_C = 1/RC$,

$$H(\omega) = \frac{1}{1+j} \cdot \frac{1-j}{1-j} = \frac{1-j}{2}$$

Magnitude (Attenuation)

$$|H(\omega)| = \left|\frac{1-j}{2}\right| = \frac{1}{\sqrt{2}}$$

$$|H(\omega)| \cong -3\text{dB}$$

Phase

$$\angle H(\omega) = \arctan\left(\frac{-1/2}{1/2}\right)$$

$$\angle H(\omega) = -45^\circ$$

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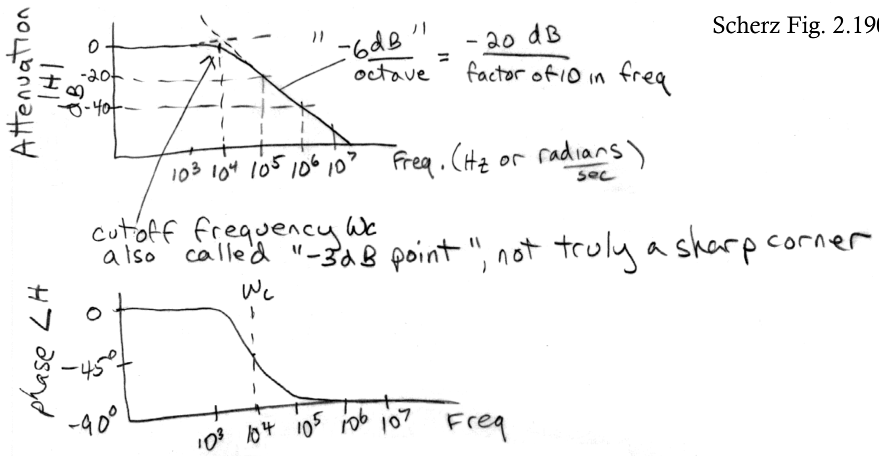
"Bode" Plot

dB Attenuation & Phase vs. log of frequency

Scherz Fig. 2.190



our low-pass filter example, with $\omega_C = 10^4$ Hz.

3 dB is factor of $\sqrt{2}$ in amplitude.
6 dB is factor of 2 in amplitude.
One *octave* is factor of 2 in frequency.

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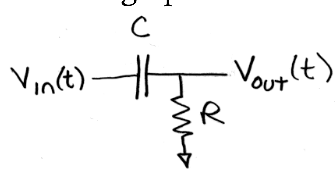
Magnitude and Phase of High-Pass Filter

Recall high-pass filter:



$$H(\omega) = \frac{V_{out}(t)}{V_{in}(t)} = \frac{j\omega RC}{1 + j\omega RC}$$

$$V_{out}(t) = H(\omega)V_{in}(t)$$

At *cut-off frequency*, $\omega_C = 1/RC$,

$$H(\omega) = \frac{j}{1+j} \cdot \frac{1-j}{1-j} = \frac{1+j}{2}$$

<u>Magnitude</u> (Attenuation)

$$|H(\omega)| = \left|\frac{1+j}{2}\right| = \frac{1}{\sqrt{2}}$$

$$|H(\omega)| \cong -3\text{dB}$$

<u>Phase</u>

$$\angle H(\omega) = \arctan\left(\frac{1/2}{1/2}\right)$$

$$\angle H(\omega) = 45°$$

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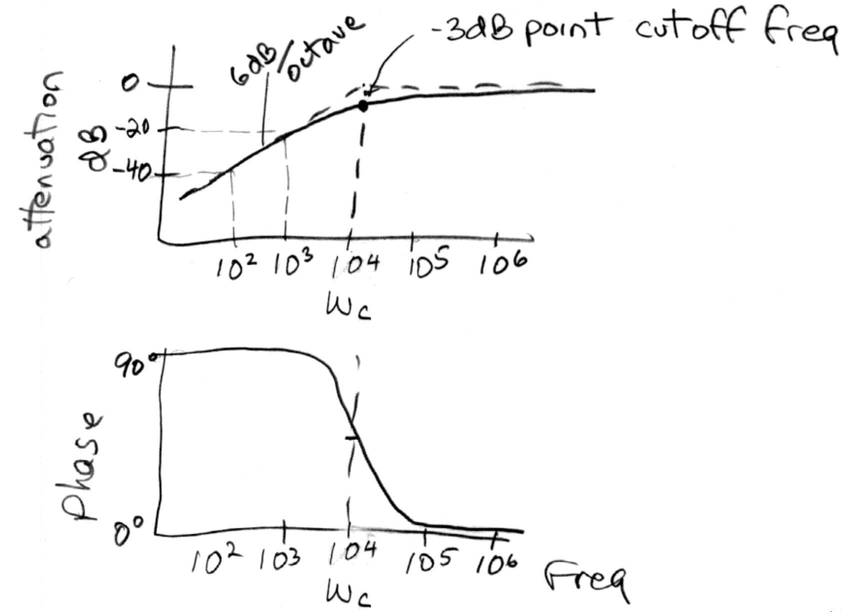
Bode Plot of High-Pass Filter



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